MULTIMEDIA UNIVERSITY

STUDENT IDENTIFICATION NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2018/2019 SESSION

BMS2024 -ADVANCED MANAGERIAL STATISTICS

(All Sections / Groups)

27 OCTOBER 2018
2.30 pm – 4.30 pm
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This question paper consists of 15 pages **excluding** the cover page.
2. This question paper consists of **FOUR** structured questions. Attempt **ALL** questions.
3. Students are allowed to use non-programmable scientific calculators with no restrictions.
4. A formulae list and statistical tables are attached at the end of the question paper.
5. Use **pen** to write the answers in the answer booklet provided.

QUESTION 1 [25 Marks]

A fast-food franchisor is considering building a restaurant at a certain location. Based on financial analyses, a location is acceptable only if the number of pedestrians passing the location is more than 100 per hour. From 40 hours of observations, the average number of pedestrians per hour was 107. Assume that the population standard deviation is known to be 16 pedestrians. Can we conclude that the proposed location is acceptable?

a) Establish the appropriate null and alternative hypotheses. [2 marks]

b) Compute the test statistic and the p-value of the test. [6 marks]

c) Based on the p-value obtained in (b), what is your statistical decision? Use $\alpha = 0.01$. [3 marks]

d) State the possible Type I and Type II errors that might occur. [4 marks]

e) At 0.05 level of significance, compute the probability of a Type II error. Given that the actual average number of pedestrians passing the location per hour is 109. [7 marks]

f) Compute the power of the test. [2 marks]

g) If the sample size is decreased, what is the effect on the power of the test. [1 mark]

Continued...

QUESTION 2 [25 Marks]

a) In a study of what motivates people to engage in sporting activity, students who regularly took part in a sport or fitness activity were asked to rate how strongly they were motivated by competition. The responses were recorded on a five-point scales; 1=weak, 2=fairly weak, 3=neutral, 4=fairly strong and 5=strong.

Determine the most appropriate statistical technique to identify any differences in their motivation in competition between three sport groups; those who regularly did athletics, played basketball and did yoga. State the reasons of your choice.

[6 marks]

b) Caffeine is a part of our daily routine. Caffeine is present naturally or added to widely consumed foods and beverages. The amounts of caffeine (mg) in a regular serving of assorted beverages are listed below.

Teas	Coffees	Cola
70	120	35
40	80	48
30	160	55
25	90	43
40	140	42

If someone wants to limit caffeine intake, does it really matter which beverage she or he chooses? At level of significance 1 percent, conduct an appropriate statistical testing method. Assume that the dataset is not normally distributed. [19 marks]

Continued...

QUESTION 3 [25 Marks]

A marketing manager of a bank claimed that the demographic profile of customers may give clear picture for the bank to understand the need of the customer including estimating their average bank balance. In selling a right product to right customer, the manager suggested to study the demographic information on customers in the bank's current market which significantly related to their bank balances.

Using banking data, he analysed three demographic profiles of 50 customers; customers' age (in years), monthly income (in RM'000) and home values (in RM'0000) to predict their average bank balances (in RM'00). The summary output of the analysis is shown below:

ANOVA

	df	*SS*	*MS*	*F*	*Significance F*
Regression	3	4341107925	1447035975	342.44	$3.065*10^{-4}$
Residual	46	194380797	4225669.499		
Total	49	7640844145			

	Coefficients	*Std Error*	*t Stat*	*P-value*
Intercept	10710.64	4260.98	2.51366	
Age	1.46	0.40691	3.58803	0.00053
Income	8.665	1.65831	5.22521	$1.011*10^{-6}$
Home value	-0.00918	0.0110	-0.831945	0.40750

a) State the multiple linear regression equation for the above data. [4 marks]

b) Interpret the slope coefficient for the customer's age and customer's monthly income relating to the bank balances. [4 marks]

c) How strong is the model? Compute and interpret the value. [4 marks]

d) At the 5 percent level of significance, test the overall validity of the model. Use the p-value approach. [4 marks]

e) At the 1 percent level of significance, test if each independent variable is significantly related towards bank balances. Use the p-value approach. [6 marks]

f) What would the customer's bank balance be if he or she is 36 years old, has monthly income around RM4,500 and owns a house with value RM 275,000? [3 marks]

Continued...

QUESTION 4 [25 Marks]

An apple juice manufacturer is planning to develop a new product; liquid concentrate. The marketing manager has to decide how to market the new product. Three strategies are considered; emphasize convenience of using the product (convenience), emphasize the quality of the product (quality) and emphasize the product's low price (price).

An experiment was conducted in three cities where advertisement campaign was launched. In each city, only one of the three marketing strategies (convenience, quality, price) was emphasized. The weekly sales (in bottles) for twenty weeks following the beginning of the campaign were recorded as below. Assume that the dataset is normally distributed.

Summary Output

Groups	Count	Sum	Mean	Variance
Convenience	20	11551	577.55	10775.00
Quality	20	13060	653.00	7238.11
Price	20	12173	608.65	8670.24

ANOVA

Source of Variation	SS	df	MS	F
Among Groups	57512	2	28756	3.23
Within Groups	506984	57	8894	
Total	564496	59		

a) What kind of ANOVA test will be appropriate for the above study? State the required conditions or assumptions for the ANOVA test to be conducted. [5 marks]

b) At the 5 percent level of significance, is there evidence of a difference in the mean weekly sales of the new product between three marketing strategies? Conduct an appropriate statistical procedure. [8 marks]

c) Conduct the Tukey-Kramer post-hoc test to examine which marketing strategies differ in mean weekly sales. Use 10 percent significance level. [12 marks]

End of Paper

STATISTICAL FORMULAE

A. DESCRIPTIVE STATISTICS

Sample Mean = $\overline{X} = \dfrac{\sum_{i=1}^{n} X_i}{n}$ Sample Standard Deviation = s = $\sqrt{\dfrac{\sum_{i=1}^{n} X_i^2}{n-1} - \dfrac{\left(\sum_{i=1}^{n} X_i\right)^2}{n(n-1)}}$

where n = *number of observations*

X_i = *the* i^{th} *observation of the data*

B. HYPOTHESIS TESTING

Types of Error
Type I Error = α= P(Rejecting H_0 | H_0 is true) where, Confidence Interval = 1 - α Type II Error = β= P(Not Rejecting H_0 | H_0 is false)

One Sample Mean Test	
σ Known	**σ Unknown**
$z = \dfrac{\overline{x} - \mu}{\sigma / \sqrt{n}}$	$t = \dfrac{\overline{x} - \mu}{s / \sqrt{n}}$
Two Sample Mean Test	
Comparing Means for Two Independent Populations	
[Standard Deviation (σ) Known] $z = \dfrac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\sigma_1^2/n_1 + \sigma_2^2/n_2}}$	**[Standard Deviation (σ) Not Known]** $t = \dfrac{(\overline{x}_1 - \overline{x}_2) - (\mu_1 - \mu_2)}{\sqrt{S_P^2\left(1/n_1 + 1/n_2\right)}}$ where $S_p^2 = \dfrac{(n_1 - 1)S_1^2 + (n_2 - 1)S_2^2}{(n_1 + n_2 - 2)}$

Two Sample Mean Test
Comparing Means for Two Paired Populations
$t = \dfrac{(\overline{D} - \mu_D)}{S_D / \sqrt{n}}$ where $\overline{D} = \dfrac{\sum_{i=1}^{n} D_i}{n}$ and $S_D = \sqrt{\dfrac{\sum_{i=1}^{n} D_i^2}{n-1} - \dfrac{\left(\sum_{i=1}^{n} D_i\right)^2}{n(n-1)}}$

Non-Parametric Analysis

Wilcoxon Rank Sum Test	Wilcoxon Signed Rank Sum Test
$Z = \frac{(T_1 - \mu_{T_1})}{\sigma_{T_1}}$ where	$Z = \frac{(T_+ - \mu_{T_+})}{\sigma_{T_+}}$ where
$\mu_{T1} = \frac{n_1(n+1)}{2}$ and	$\mu_{T+} = \frac{n(n+1)}{4}$ and
$\sigma_{T_1} = \sqrt{\frac{n_1 n_2 (n+1)}{12}}$ where $n = n_1 + n_2$	$\sigma_{T_+} = \sqrt{\frac{n(n+1)(2n+1)}{24}}$

Kruskal-Wallis Rank Test

$$H = \left[\frac{12}{n(n+1)}\sum_{j=1}^{c}\frac{T_j^2}{n_j}\right] - 3(n+1)$$ where the critical value is χ^2 with $df = c - 1$

Check ranking sum: $\sum T_j = n(n+1)/2$

Chi-Square Test

$$\chi^2 = \sum^{n}\frac{(O-E)^2}{E}$$

where O = Frequency of Observed Values and E = Frequency of Expected Values

with $df = c - 1$ *where c = number of categories*

or

with $df = (r-1)(c-1)$ *where r = number of rows and c = number of columns*

C. ANALYSIS OF VARIANCE (ANOVA)

One-Way ANOVA

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F-statistic
Among Groups	$c - 1$	SSA	MSA = SSA/c-1	MSA/MSW
Within Groups	$n - c$	SSW	MSW = SSW/n-c	
Total	$n - 1$	SST		

$$SST = \sum_{j=1}^{c}\sum_{i=1}^{n_j}\left(X_{ij} - \overline{\overline{X}}\right)^2$$ *or alternative formula:*

$$SST = \left(\sum_{j=1}^{c}\sum_{i=1}^{n_j}X_{ij}^2\right) - \frac{\left(\sum_{j=1}^{c}\sum_{i=1}^{n_j}X_{ij}\right)^2}{n}$$

$$SSA = \sum_{j=1}^{c} n_j\left(\overline{X}_j - \overline{\overline{X}}\right)^2$$ *and SSW = SST - SSA*

where n = number of observations, c = number of groups and $\overline{\overline{X}}$ *= overall mean*

Tukey-Kramer Procedure

$$\text{Critical Range} = Q_U \sqrt{\frac{MSW}{2}\left[\frac{1}{n_i}+\frac{1}{n_j}\right]}$$

where Q_u = the upper tail critical value from a Studentized Range Distribution having (c) degrees of freedom in the numerator and (n – c) degrees of freedom in the denominator at a given level of significance

Two-Way ANOVA

Source	Degrees of Freedom	Sum of Squares	Mean Squares	F-statistic
A	$r-1$	SSA	MSA = SSA/$(r-1)$	MSA / MSE
B	$c-1$	SSB	MSB = SSB/$(c-1)$	MSB / MSE
AB	$(r-1)(c-1)$	SSAB	MSAB = SSAB/$(r-1)(c-1)$	MSAB / MSE
Error	$rc(n-1)$	SSE	MSE = SSE/$rc(n'-1)$	
Total	$n-1$	SST		

where,

Factor A levels are represented by the rows and Factor B levels are represented by the columns and
n = number of observations
c = number of columns
r = number of rows
n' = number of replicates

$$SST = \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n'}\left(X_{ijk}-\overline{\overline{X}}\right)^2 \qquad SSA = cn'\sum_{i=1}^{r}\left(\overline{X}_i-\overline{\overline{X}}\right)^2$$

$$SSB = rn'\sum_{j=1}^{c}\left(\overline{X}j-\overline{\overline{X}}\right)^2 \qquad \text{where } \overline{\overline{X}} = \text{overall mean}$$

$$SSE = (n'-1)\left[S_1^2+S_2^2+\dots\dots+S_k^2\right] \qquad \text{where } S_i^2 = \text{variance of each block}$$

D. REGRESSION ANALYSIS

Multiple Linear Regression

Population Model: $Y = \beta_0+\beta_1X_1+\beta_2X_2+\dots\dots+\beta_kX_k+\varepsilon$

Sample Model: $\hat{y} = b_0+b_1x_1+b_2x_2+\dots\dots+b_kx_k+e$

Adjusted R-Square $= 1-\left[\frac{(1-R^2)(n-1)}{(n-p-1)}\right]$ where p = number of independent/predictor variables

ANOVA Table for Regression			
Source	**Degrees of Freedom**	**Sum of Squares**	**Mean Squares**
Regression	p	SSR	MSR = SSR/p
Error/Residual	$n-p-1$	SSE	MSE = SSE/$(n-p-1)$
Total	$n-1$	SST	

Test Statistic for Significance of the Overall Regression Model

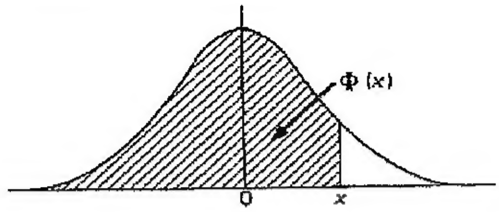
F = MSR/MSE

Test Statistic for Significance of Each Predictor Variable

$t_i = \frac{b_i}{S_{b_i}}$ and the critical value $= \pm t_{\alpha/2,(n-p-1)}$

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

The function tabulated is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2}\,dt$. $\Phi(x)$ is the probability that a random variable, normally distributed with zero mean and unit variance, will be less than or equal to x. When $x < 0$ use $\Phi(x) = 1 - \Phi(-x)$, as the normal distribution with zero mean and unit variance is symmetric about zero.



x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
0·00	0·5000	0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·97725
·01	·5040	·41	·6591	·81	·7910	·21	·8869	·61	·9463	·01	·97778
·02	·5080	·42	·6628	·82	·7939	·22	·8888	·62	·9474	·02	·97831
·03	·5120	·43	·6664	·83	·7967	·23	·8907	·63	·9484	·03	·97882
·04	·5160	·44	·6700	·84	·7995	·24	·8925	·64	·9495	·04	·97932
0·05	0·5199	0·45	0·6736	0·85	0·8023	1·25	0·8944	1·65	0·9505	2·05	0·97982
·06	·5239	·46	·6772	·86	·8051	·26	·8962	·66	·9515	·06	·98030
·07	·5279	·47	·6808	·87	·8078	·27	·8980	·67	·9525	·07	·98077
·08	·5319	·48	·6844	·88	·8106	·28	·8997	·68	·9535	·08	·98124
·09	·5359	·49	·6879	·89	·8133	·29	·9015	·69	·9545	·09	·98169
0·10	0·5398	0·50	0·6915	0·90	0·8159	1·30	0·9032	1·70	0·9554	2·10	0·98214
·11	·5438	·51	·6950	·91	·8186	·31	·9049	·71	·9564	·11	·98257
·12	·5478	·52	·6985	·92	·8212	·32	·9066	·72	·9573	·12	·98300
·13	·5517	·53	·7019	·93	·8238	·33	·9082	·73	·9582	·13	·98341
·14	·5557	·54	·7054	·94	·8264	·34	·9099	·74	·9591	·14	·98382
0·15	0·5596	0·55	0·7088	0·95	0·8289	1·35	0·9115	1·75	0·9599	2·15	0·98422
·16	·5636	·56	·7123	·96	·8315	·36	·9131	·76	·9608	·16	·98461
·17	·5675	·57	·7157	·97	·8340	·37	·9147	·77	·9616	·17	·98500
·18	·5714	·58	·7190	·98	·8365	·38	·9162	·78	·9625	·18	·98537
·19	·5753	·59	·7224	·99	·8389	·39	·9177	·79	·9633	·19	·98574
0·20	0·5793	0·60	0·7257	1·00	0·8413	1·40	0·9192	1·80	0·9641	2·20	0·98610
·21	·5832	·61	·7291	·01	·8438	·41	·9207	·81	·9649	·21	·98645
·22	·5871	·62	·7324	·02	·8461	·42	·9222	·82	·9656	·22	·98679
·23	·5910	·63	·7357	·03	·8485	·43	·9236	·83	·9664	·23	·98713
·24	·5948	·64	·7389	·04	·8508	·44	·9251	·84	·9671	·24	·98745
0·25	0·5987	0·65	0·7422	1·05	0·8531	1·45	0·9265	1·85	0·9678	2·25	0·98778
·26	·6026	·66	·7454	·06	·8554	·46	·9279	·86	·9686	·26	·98809
·27	·6064	·67	·7486	·07	·8577	·47	·9292	·87	·9693	·27	·98840
·28	·6103	·68	·7517	·08	·8599	·48	·9306	·88	·9699	·28	·98870
·29	·6141	·69	·7549	·09	·8621	·49	·9319	·89	·9706	·29	·98899
0·30	0·6179	0·70	0·7580	1·10	0·8643	1·50	0·9332	1·90	0·9713	2·30	0·98928
·31	·6217	·71	·7611	·11	·8665	·51	·9345	·91	·9719	·31	·98956
·32	·6255	·72	·7642	·12	·8686	·52	·9357	·92	·9726	·32	·98983
·33	·6293	·73	·7673	·13	·8708	·53	·9370	·93	·9732	·33	·99010
·34	·6331	·74	·7704	·14	·8729	·54	·9382	·94	·9738	·34	·99036
0·35	0·6368	0·75	0·7734	1·15	0·8749	1·55	0·9394	1·95	0·9744	2·35	0·99061
·36	·6406	·76	·7764	·16	·8770	·56	·9406	·96	·9750	·36	·99086
·37	·6443	·77	·7794	·17	·8790	·57	·9418	·97	·9756	·37	·99111
·38	·6480	·78	·7823	·18	·8810	·58	·9429	·98	·9761	·38	·99134
·39	·6517	·79	·7852	·19	·8830	·59	·9441	·99	·9767	·39	·99158
0·40	0·6554	0·80	0·7881	1·20	0·8849	1·60	0·9452	2·00	0·9772	2·40	0·99180

TABLE 4. THE NORMAL DISTRIBUTION FUNCTION

x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$	x	$\Phi(x)$
2·40	0·99180	2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918
·41	·99202	·56	·99477	·71	·99664	·86	·99788	·01	·99869	·16	·99921
·42	·99224	·57	·99492	·72	·99674	·87	·99795	·02	·99874	·17	·99924
·43	·99245	·58	·99506	·73	·99683	·88	·99801	·03	·99878	·18	·99926
·44	·99266	·59	·99520	·74	·99693	·89	·99807	·04	·99882	·19	·99929
2·45	0·99286	2·60	0·99534	2·75	0·99702	2·90	0·99813	3·05	0·99886	3·20	0·99931
·46	·99305	·61	·99547	·76	·99711	·91	·99819	·06	·99889	·21	·99934
·47	·99324	·62	·99560	·77	·99720	·92	·99825	·07	·99893	·22	·99936
·48	·99343	·63	·99573	·78	·99728	·93	·99831	·08	·99896	·23	·99938
·49	·99361	·64	·99585	·79	·99736	·94	·99836	·09	·99900	·24	·99940
2·50	0·99379	2·65	0·99598	2·80	0·99744	2·95	0·99841	3·10	0·99903	3·25	0·99942
·51	·99396	·66	·99609	·81	·99752	·96	·99846	·11	·99906	·26	·99944
·52	·99413	·67	·99621	·82	·99760	·97	·99851	·12	·99910	·27	·99946
·53	·99430	·68	·99632	·83	·99767	·98	·99856	·13	·99913	·28	·99948
·54	·99446	·69	·99643	·84	·99774	·99	·99861	·14	·99916	·29	·99950
2·55	0·99461	2·70	0·99653	2·85	0·99781	3·00	0·99865	3·15	0·99918	3·30	0·99952

The critical table below gives on the left the range of values of x for which $\Phi(x)$ takes the value on the right, correct to the last figure given; in critical cases, take the upper of the two values of $\Phi(x)$ indicated.

			0·9994		0·99990		0·99995
3·075		3·263		3·731		3·916	
	0·9990		0·9995		0·99991		0·99996
3·105		3·320		3·759		3·976	
	0·9991		0·9996		0·99992		0·99997
3·138		3·389		3·791		4·055	
	0·9992		0·9997		0·99993		0·99998
3·174		3·480		3·826		4·173	
	0·9993		0·9998		0·99994		0·99999
3·215		3·615		3·867		4·417	
	0·9994		0·9999		0·99995		1·00000

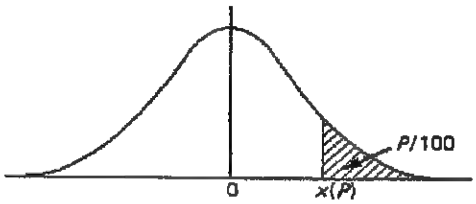
When $x > 3{\cdot}3$ the formula $1-\Phi(x) \doteqdot \frac{e^{-\frac{1}{2}x^2}}{x\sqrt{2\pi}}\left[1-\frac{1}{x^2}+\frac{3}{x^4}-\frac{15}{x^6}+\frac{105}{x^8}\right]$ is very accurate, with relative error less than $945/x^{10}$.

TABLE 5. PERCENTAGE POINTS OF THE NORMAL DISTRIBUTION

This table gives percentage points $x(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{2\pi}}\int_{x(P)}^{\infty} e^{-\frac{1}{2}t^2}\,dt.$$

If X is a variable, normally distributed with zero mean and unit variance, $P/100$ is the probability that $X \geqslant x(P)$. The lower P per cent points are given by symmetry as $-x(P)$, and the probability that $|X| \geqslant x(P)$ is $2P/100$.



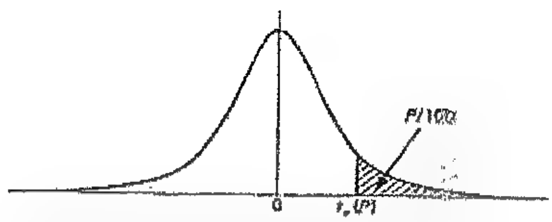
P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$	P	$x(P)$
50	0·0000	5·0	1·6449	3·0	1·8808	2·0	2·0537	1·0	2·3263	0·10	3·0902
45	0·1257	4·8	1·6646	2·9	1·8957	1·9	2·0749	0·9	2·3656	0·09	3·1214
40	0·2533	4·6	1·6849	2·8	1·9110	1·8	2·0969	0·8	2·4089	0·08	3·1559
35	0·3853	4·4	1·7060	2·7	1·9268	1·7	2·1201	0·7	2·4573	0·07	3·1947
30	0·5244	4·2	1·7279	2·6	1·9431	1·6	2·1444	0·6	2·5121	0·06	3·2389
25	0·6745	4·0	1·7507	2·5	1·9600	1·5	2·1701	0·5	2·5758	0·05	3·2905
20	0·8416	3·8	1·7744	2·4	1·9774	1·4	2·1973	0·4	2·6521	0·01	3·7190
15	1·0364	3·6	1·7991	2·3	1·9954	1·3	2·2262	0·3	2·7478	0·005	3·8906
10	1·2816	3·4	1·8250	2·2	2·0141	1·2	2·2571	0·2	2·8782	0·001	4·2649
5	1·6449	3·2	1·8522	2·1	2·0335	1·1	2·2904	0·1	3·0902	0·0005	4·4172

TABLE 10. PERCENTAGE POINTS OF THE t-DISTRIBUTION

This table gives percentage points $t_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{\sqrt{\nu\pi}}\,\frac{\Gamma(\frac{1}{2}\nu+\frac{1}{2})}{\Gamma(\frac{1}{2}\nu)}\int_{t_\nu(P)}^{\infty}\frac{dt}{(1+t^2/\nu)^{\frac{1}{2}(\nu+1)}}.$$

Let X_1 and X_2 be independent random variables having a normal distribution with zero mean and unit variance and a χ^2-distribution with ν degrees of freedom respectively; then $t = X_1/\sqrt{X_2/\nu}$ has Student's t-distribution with ν degrees of freedom, and the probability that $t \geqslant t_\nu(P)$ is $P/100$. The lower percentage points are given by symmetry as $-t_\nu(P)$, and the probability that $|t| \geqslant t_\nu(P)$ is $2P/100$.



The limiting distribution of t as ν tends to infinity is the normal distribution with zero mean and unit variance. When ν is large interpolation in ν should be harmonic.

P	40	30	25	20	15	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·3249	0·7265	1·0000	1·3764	1·963	3·078	6·314	12·71	31·82	63·66	318·3	636·6
2	0·2887	0·6172	0·8165	1·0607	1·386	1·886	2·920	4·303	6·965	9·925	22·33	31·60
3	0·2767	0·5844	0·7649	0·9785	1·250	1·638	2·353	3·182	4·541	5·841	10·21	12·92
4	0·2707	0·5686	0·7407	0·9410	1·190	1·533	2·132	2·776	3·747	4·604	7·173	8·610
5	0·2672	0·5594	0·7267	0·9195	1·156	1·476	2·015	2·571	3·365	4·032	5·893	6·869
6	0·2648	0·5534	0·7176	0·9057	1·134	1·440	1·943	2·447	3·143	3·707	5·208	5·959
7	0·2632	0·5491	0·7111	0·8960	1·119	1·415	1·895	2·365	2·998	3·499	4·785	5·408
8	0·2619	0·5459	0·7064	0·8889	1·108	1·397	1·860	2·306	2·896	3·355	4·501	5·041
9	0·2610	0·5435	0·7027	0·8834	1·100	1·383	1·833	2·262	2·821	3·250	4·297	4·781
10	0·2602	0·5415	0·6998	0·8791	1·093	1·372	1·812	2·228	2·764	3·169	4·144	4·587
11	0·2596	0·5399	0·6974	0·8755	1·088	1·363	1·796	2·201	2·718	3·106	4·025	4·437
12	0·2590	0·5386	0·6955	0·8726	1·083	1·356	1·782	2·179	2·681	3·055	3·930	4·318
13	0·2586	0·5375	0·6938	0·8702	1·079	1·350	1·771	2·160	2·650	3·012	3·852	4·221
14	0·2582	0·5366	0·6924	0·8681	1·076	1·345	1·761	2·145	2·624	2·977	3·787	4·140
15	0·2579	0·5357	0·6912	0·8662	1·074	1·341	1·753	2·131	2·602	2·947	3·733	4·073
16	0·2576	0·5350	0·6901	0·8647	1·071	1·337	1·746	2·120	2·583	2·921	3·686	4·015
17	0·2573	0·5344	0·6892	0·8633	1·069	1·333	1·740	2·110	2·567	2·898	3·646	3·965
18	0·2571	0·5338	0·6884	0·8620	1·067	1·330	1·734	2·101	2·552	2·878	3·610	3·922
19	0·2569	0·5333	0·6876	0·8610	1·066	1·328	1·729	2·093	2·539	2·861	3·579	3·883
20	0·2567	0·5329	0·6870	0·8600	1·064	1·325	1·725	2·086	2·528	2·845	3·552	3·850
21	0·2566	0·5325	0·6864	0·8591	1·063	1·323	1·721	2·080	2·518	2·831	3·527	3·819
22	0·2564	0·5321	0·6858	0·8583	1·061	1·321	1·717	2·074	2·508	2·819	3·505	3·792
23	0·2563	0·5317	0·6853	0·8575	1·060	1·319	1·714	2·069	2·500	2·807	3·485	3·768
24	0·2562	0·5314	0·6848	0·8569	1·059	1·318	1·711	2·064	2·492	2·797	3·467	3·745
25	0·2561	0·5312	0·6844	0·8562	1·058	1·316	1·708	2·060	2·485	2·787	3·450	3·725
26	0·2560	0·5309	0·6840	0·8557	1·058	1·315	1·706	2·056	2·479	2·779	3·435	3·707
27	0·2559	0·5306	0·6837	0·8551	1·057	1·314	1·703	2·052	2·473	2·771	3·421	3·690
28	0·2558	0·5304	0·6834	0·8546	1·056	1·313	1·701	2·048	2·467	2·763	3·408	3·674
29	0·2557	0·5302	0·6830	0·8542	1·055	1·311	1·699	2·045	2·462	2·756	3·396	3·659
30	0·2556	0·5300	0·6828	0·8538	1·055	1·310	1·697	2·042	2·457	2·750	3·385	3·646
32	0·2555	0·5297	0·6822	0·8530	1·054	1·309	1·694	2·037	2·449	2·738	3·365	3·622
34	0·2553	0·5294	0·6818	0·8523	1·052	1·307	1·691	2·032	2·441	2·728	3·348	3·601
36	0·2552	0·5291	0·6814	0·8517	1·052	1·306	1·688	2·028	2·434	2·719	3·333	3·582
38	0·2551	0·5288	0·6810	0·8512	1·051	1·304	1·686	2·024	2·429	2·712	3·319	3·566
40	0·2550	0·5286	0·6807	0·8507	1·050	1·303	1·684	2·021	2·423	2·704	3·307	3·551
50	0·2547	0·5278	0·6794	0·8489	1·047	1·299	1·676	2·009	2·403	2·678	3·261	3·496
60	0·2545	0·5272	0·6786	0·8477	1·045	1·296	1·671	2·000	2·390	2·660	3·232	3·460
120	0·2539	0·5258	0·6765	0·8446	1·041	1·289	1·658	1·980	2·358	2·617	3·160	3·373
∞	0·2533	0·5244	0·6745	0·8416	1·036	1·282	1·645	1·960	2·326	2·576	3·090	3·291

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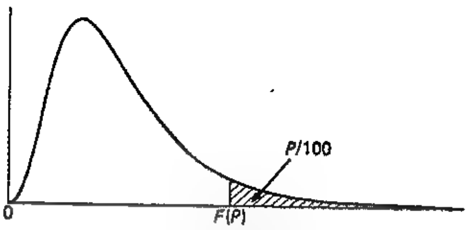
TABLE 12(a). 10 PER CENT POINTS OF THE *F*-DISTRIBUTION

The function tabulated is $F(P) = F(P|\nu_1, \nu_2)$ defined by the equation

$$\frac{P}{100} = \frac{\Gamma(\frac{1}{2}\nu_1+\frac{1}{2}\nu_2)}{\Gamma(\frac{1}{2}\nu_1)\,\Gamma(\frac{1}{2}\nu_2)}\,\nu_1^{\frac{1}{2}\nu_1}\,\nu_2^{\frac{1}{2}\nu_2}\int_{F(P)}^{\infty}\frac{F^{\frac{1}{2}\nu_1-1}}{(\nu_2+\nu_1 F)^{\frac{1}{2}(\nu_1+\nu_2)}}\,dF,$$

for $P = 10$, 5, 2·5, 1, 0·5 and 0·1. The lower percentage points, that is the values $F'(P) = F'(P|\nu_1, \nu_2)$ such that the probability that $F \leqslant F'(P)$ is equal to $P/100$, may be found by the formula

$$F'(P|\nu_1, \nu_2) = 1/F(P|\nu_2, \nu_1).$$

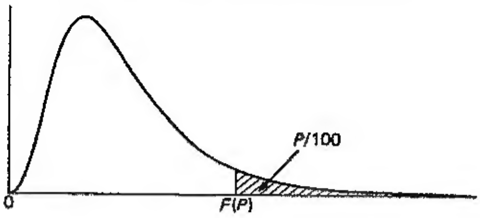


(This shape applies only when $\nu_1 \geqslant 3$. When $\nu_1 < 3$ the mode is at the origin.)

$\nu_1 =$	1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2 =$ 1	39.86	49.50	53.59	55.83	57.24	58.20	58.91	59.44	60.19	60.71	62.00	63.33
2	8.526	9.000	9.162	9.243	9.293	9.326	9.349	9.367	9.392	9.408	9.450	9.491
3	5.538	5.462	5.391	5.343	5.309	5.285	5.266	5.252	5.230	5.216	5.176	5.134
4	4.545	4.325	4.191	4.107	4.051	4.010	3.979	3.955	3.920	3.896	3.831	3.761
5	4.060	3.780	3.619	3.520	3.453	3.405	3.368	3.339	3.297	3.268	3.191	3.105
6	3.776	3.463	3.289	3.181	3.108	3.055	3.014	2.983	2.937	2.905	2.818	2.722
7	3.589	3.257	3.074	2.961	2.883	2.827	2.785	2.752	2.703	2.668	2.575	2.471
8	3.458	3.113	2.924	2.806	2.726	2.668	2.624	2.589	2.538	2.502	2.404	2.293
9	3.360	3.006	2.813	2.693	2.611	2.551	2.505	2.469	2.416	2.379	2.277	2.159
10	3.285	2.924	2.728	2.605	2.522	2.461	2.414	2.377	2.323	2.284	2.178	2.055
11	3.225	2.860	2.660	2.536	2.451	2.389	2.342	2.304	2.248	2.209	2.100	1.972
12	3.177	2.807	2.606	2.480	2.394	2.331	2.283	2.245	2.188	2.147	2.036	1.904
13	3.136	2.763	2.560	2.434	2.347	2.283	2.234	2.195	2.138	2.097	1.983	1.846
14	3.102	2.726	2.522	2.395	2.307	2.243	2.193	2.154	2.095	2.054	1.938	1.797
15	3.073	2.695	2.490	2.361	2.273	2.208	2.158	2.119	2.059	2.017	1.899	1.755
16	3.048	2.668	2.462	2.333	2.244	2.178	2.128	2.088	2.028	1.985	1.866	1.718
17	3.026	2.645	2.437	2.308	2.218	2.152	2.102	2.061	2.001	1.958	1.836	1.686
18	3.007	2.624	2.416	2.286	2.196	2.130	2.079	2.038	1.977	1.933	1.810	1.657
19	2.990	2.606	2.397	2.266	2.176	2.109	2.058	2.017	1.956	1.912	1.787	1.631
20	2.975	2.589	2.380	2.249	2.158	2.091	2.040	1.999	1.937	1.892	1.767	1.607
21	2.961	2.575	2.365	2.233	2.142	2.075	2.023	1.982	1.920	1.875	1.748	1.586
22	2.949	2.561	2.351	2.219	2.128	2.060	2.008	1.967	1.904	1.859	1.731	1.567
23	2.937	2.549	2.339	2.207	2.115	2.047	1.995	1.953	1.890	1.845	1.716	1.549
24	2.927	2.538	2.327	2.195	2.103	2.035	1.983	1.941	1.877	1.832	1.702	1.533
25	2.918	2.528	2.317	2.184	2.092	2.024	1.971	1.929	1.866	1.820	1.689	1.518
26	2.909	2.519	2.307	2.174	2.082	2.014	1.961	1.919	1.855	1.809	1.677	1.504
27	2.901	2.511	2.299	2.165	2.073	2.005	1.952	1.909	1.845	1.799	1.666	1.491
28	2.894	2.503	2.291	2.157	2.064	1.996	1.943	1.900	1.836	1.790	1.656	1.478
29	2.887	2.495	2.283	2.149	2.057	1.988	1.935	1.892	1.827	1.781	1.647	1.467
30	2.881	2.489	2.276	2.142	2.049	1.980	1.927	1.884	1.819	1.773	1.638	1.456
32	2.869	2.477	2.263	2.129	2.036	1.967	1.913	1.870	1.805	1.758	1.622	1.437
34	2.859	2.466	2.252	2.118	2.024	1.955	1.901	1.858	1.793	1.745	1.608	1.419
36	2.850	2.456	2.243	2.108	2.014	1.945	1.891	1.847	1.781	1.734	1.595	1.404
38	2.842	2.448	2.234	2.099	2.005	1.935	1.881	1.838	1.772	1.724	1.584	1.390
40	2.835	2.440	2.226	2.091	1.997	1.927	1.873	1.829	1.763	1.715	1.574	1.377
60	2.791	2.393	2.177	2.041	1.946	1.875	1.819	1.775	1.707	1.657	1.511	1.291
120	2.748	2.347	2.130	1.992	1.896	1.824	1.767	1.722	1.652	1.601	1.447	1.193
∞	2.706	2.303	2.084	1.945	1.847	1.774	1.717	1.670	1.599	1.546	1.383	1.000

TABLE 12(*b*). 5 PER CENT POINTS OF THE *F*-DISTRIBUTION

If $F = \frac{X_1}{\nu_1} \Big/ \frac{X_2}{\nu_2}$, where X_1 and X_2 are independent random variables distributed as χ^2 with ν_1 and ν_2 degrees of freedom respectively, then the probabilities that $F \geqslant F(P)$ and that $F \leqslant F'(P)$ are both equal to $P/100$. Linear interpolation in ν_1 and ν_2 will generally be sufficiently accurate except when either $\nu_1 > 12$ or $\nu_2 > 40$, when harmonic interpolation should be used.



(This shape applies only when $\nu_1 \geqslant 3$. When $\nu_1 < 3$ the mode is at the origin.)

$\nu_1 =$	1	2	3	4	5	6	7	8	10	12	24	∞
$\nu_2 =$ 1	161·4	199·5	215·7	224·6	230·2	234·0	236·8	238·9	241·9	243·9	249·1	254·3
2	18·51	19·00	19·16	19·25	19·30	19·33	19·35	19·37	19·40	19·41	19·45	19·50
3	10·13	9·552	9·277	9·117	9·013	8·941	8·887	8·845	8·786	8·745	8·639	8·526
4	7·709	6·944	6·591	6·388	6·256	6·163	6·094	6·041	5·964	5·912	5·774	5·628
5	6·608	5·786	5·409	5·192	5·050	4·950	4·876	4·818	4·735	4·678	4·527	4·365
6	5·987	5·143	4·757	4·534	4·387	4·284	4·207	4·147	4·060	4·000	3·841	3·669
7	5·591	4·737	4·347	4·120	3·972	3·866	3·787	3·726	3·637	3·575	3·410	3·230
8	5·318	4·459	4·066	3·838	3·687	3·581	3·500	3·438	3·347	3·284	3·115	2·928
9	5·117	4·256	3·863	3·633	3·482	3·374	3·293	3·230	3·137	3·073	2·900	2·707
10	4·965	4·103	3·708	3·478	3·326	3·217	3·135	3·072	2·978	2·913	2·737	2·538
11	4·844	3·982	3·587	3·357	3·204	3·095	3·012	2·948	2·854	2·788	2·609	2·404
12	4·747	3·885	3·490	3·259	3·106	2·996	2·913	2·849	2·753	2·687	2·505	2·296
13	4·667	3·806	3·411	3·179	3·025	2·915	2·832	2·767	2·671	2·604	2·420	2·206
14	4·600	3·739	3·344	3·112	2·958	2·848	2·764	2·699	2·602	2·534	2·349	2·131
15	4·543	3·682	3·287	3·056	2·901	2·790	2·707	2·641	2·544	2·475	2·288	2·066
16	4·494	3·634	3·239	3·007	2·852	2·741	2·657	2·591	2·494	2·425	2·235	2·010
17	4·451	3·592	3·197	2·965	2·810	2·699	2·614	2·548	2·450	2·381	2·190	1·960
18	4·414	3·555	3·160	2·928	2·773	2·661	2·577	2·510	2·412	2·342	2·150	1·917
19	4·381	3·522	3·127	2·895	2·740	2·628	2·544	2·477	2·378	2·308	2·114	1·878
20	4·351	3·493	3·098	2·866	2·711	2·599	2·514	2·447	2·348	2·278	2·082	1·843
21	4·325	3·467	3·072	2·840	2·685	2·573	2·488	2·420	2·321	2·250	2·054	1·812
22	4·301	3·443	3·049	2·817	2·661	2·549	2·464	2·397	2·297	2·226	2·028	1·783
23	4·279	3·422	3·028	2·796	2·640	2·528	2·442	2·375	2·275	2·204	2·005	1·757
24	4·260	3·403	3·009	2·776	2·621	2·508	2·423	2·355	2·255	2·183	1·984	1·733
25	4·242	3·385	2·991	2·759	2·603	2·490	2·405	2·337	2·236	2·165	1·964	1·711
26	4·225	3·369	2·975	2·743	2·587	2·474	2·388	2·321	2·220	2·148	1·946	1·691
27	4·210	3·354	2·960	2·728	2·572	2·459	2·373	2·305	2·204	2·132	1·930	1·672
28	4·196	3·340	2·947	2·714	2·558	2·445	2·359	2·291	2·190	2·118	1·915	1·654
29	4·183	3·328	2·934	2·701	2·545	2·432	2·346	2·278	2·177	2·104	1·901	1·638
30	4·171	3·316	2·922	2·690	2·534	2·421	2·334	2·266	2·165	2·092	1·887	1·622
32	4·149	3·295	2·901	2·668	2·512	2·399	2·313	2·244	2·142	2·070	1·864	1·594
34	4·130	3·276	2·883	2·650	2·494	2·380	2·294	2·225	2·123	2·050	1·843	1·569
36	4·113	3·259	2·866	2·634	2·477	2·364	2·277	2·209	2·106	2·033	1·824	1·547
38	4·098	3·245	2·852	2·619	2·463	2·349	2·262	2·194	2·091	2·017	1·808	1·527
40	4·085	3·232	2·839	2·606	2·449	2·336	2·249	2·180	2·077	2·003	1·793	1·509
60	4·001	3·150	2·758	2·525	2·368	2·254	2·167	2·097	1·993	1·917	1·700	1·389
120	3·920	3·072	2·680	2·447	2·290	2·175	2·087	2·016	1·910	1·834	1·608	1·254
∞	3·841	2·996	2·605	2·372	2·214	2·099	2·010	1·938	1·831	1·752	1·517	1·000

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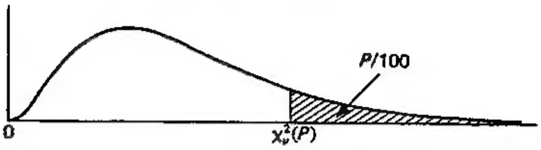
TABLE 8. PERCENTAGE POINTS OF THE χ^2-DISTRIBUTION

This table gives percentage points $\chi^2_\nu(P)$ defined by the equation

$$\frac{P}{100} = \frac{1}{2^{\nu/2}\,\Gamma(\frac{\nu}{2})} \int_{\chi^2_\nu(P)}^{\infty} x^{\frac{1}{2}\nu-1} e^{-\frac{1}{2}x}\, dx.$$

If X is a variable distributed as χ^2 with ν degrees of freedom, $P/100$ is the probability that $X \geqslant \chi^2_\nu(P)$.

For $\nu > 100$, $\sqrt{2X}$ is approximately normally distributed with mean $\sqrt{2\nu-1}$ and unit variance.



(The above shape applies for $\nu \geqslant 3$ only. When $\nu < 3$ the mode is at the origin.)

P	50	40	30	20	10	5	2·5	1	0·5	0·1	0·05
$\nu = 1$	0·4549	0·7083	1·074	1·642	2·706	3·841	5·024	6·635	7·879	10·83	12·12
2	1·386	1·833	2·408	3·219	4·605	5·991	7·378	9·210	10·60	13·82	15·20
3	2·366	2·946	3·665	4·642	6·251	7·815	9·348	11·34	12·84	16·27	17·73
4	3·357	4·045	4·878	5·989	7·779	9·488	11·14	13·28	14·86	18·47	20·00
5	4·351	5·132	6·064	7·289	9·236	11·07	12·83	15·09	16·75	20·52	22·11
6	5·348	6·211	7·231	8·558	10·64	12·59	14·45	16·81	18·55	22·46	24·10
7	6·346	7·283	8·383	9·803	12·02	14·07	16·01	18·48	20·28	24·32	26·02
8	7·344	8·351	9·524	11·03	13·36	15·51	17·53	20·09	21·95	26·12	27·87
9	8·343	9·414	10·66	12·24	14·68	16·92	19·02	21·67	23·59	27·88	29·67
10	9·342	10·47	11·78	13·44	15·99	18·31	20·48	23·21	25·19	29·59	31·42
11	10·34	11·53	12·90	14·63	17·28	19·68	21·92	24·72	26·76	31·26	33·14
12	11·34	12·58	14·01	15·81	18·55	21·03	23·34	26·22	28·30	32·91	34·82
13	12·34	13·64	15·12	16·98	19·81	22·36	24·74	27·69	29·82	34·53	36·48
14	13·34	14·69	16·22	18·15	21·06	23·68	26·12	29·14	31·32	36·12	38·11
15	14·34	15·73	17·32	19·31	22·31	25·00	27·49	30·58	32·80	37·70	39·72
16	15·34	16·78	18·42	20·47	23·54	26·30	28·85	32·00	34·27	39·25	41·31
17	16·34	17·82	19·51	21·61	24·77	27·59	30·19	33·41	35·72	40·79	42·88
18	17·34	18·87	20·60	22·76	25·99	28·87	31·53	34·81	37·16	42·31	44·43
19	18·34	19·91	21·69	23·90	27·20	30·14	32·85	36·19	38·58	43·82	45·97
20	19·34	20·95	22·77	25·04	28·41	31·41	34·17	37·57	40·00	45·31	47·50
21	20·34	21·99	23·86	26·17	29·62	32·67	35·48	38·93	41·40	46·80	49·01
22	21·34	23·03	24·94	27·30	30·81	33·92	36·78	40·29	42·80	48·27	50·51
23	22·34	24·07	26·02	28·43	32·01	35·17	38·08	41·64	44·18	49·73	52·00
24	23·34	25·11	27·10	29·55	33·20	36·42	39·36	42·98	45·56	51·18	53·48
25	24·34	26·14	28·17	30·68	34·38	37·65	40·65	44·31	46·93	52·62	54·95
26	25·34	27·18	29·25	31·79	35·56	38·89	41·92	45·64	48·29	54·05	56·41
27	26·34	28·21	30·32	32·91	36·74	40·11	43·19	46·96	49·64	55·48	57·86
28	27·34	29·25	31·39	34·03	37·92	41·34	44·46	48·28	50·99	56·89	59·30
29	28·34	30·28	32·46	35·14	39·09	42·56	45·72	49·59	52·34	58·30	60·73
30	29·34	31·32	33·53	36·25	40·26	43·77	46·98	50·89	53·67	59·70	62·16
32	31·34	33·38	35·66	38·47	42·58	46·19	49·48	53·49	56·33	62·49	65·00
34	33·34	35·44	37·80	40·68	44·90	48·60	51·97	56·06	58·96	65·25	67·80
36	35·34	37·50	39·92	42·88	47·21	51·00	54·44	58·62	61·58	67·99	70·59
38	37·34	39·56	42·05	45·08	49·51	53·38	56·90	61·16	64·18	70·70	73·35
40	39·34	41·62	44·16	47·27	51·81	55·76	59·34	63·69	66·77	73·40	76·09
50	49·33	51·89	54·72	58·16	63·17	67·50	71·42	76·15	79·49	86·66	89·56
60	59·33	62·13	65·23	68·97	74·40	79·08	83·30	88·38	91·95	99·61	102·7
70	69·33	72·36	75·69	79·71	85·53	90·53	95·02	100·4	104·2	112·3	115·6
80	79·33	82·57	86·12	90·41	96·58	101·9	106·6	112·3	116·3	124·8	128·3
90	89·33	92·76	96·52	101·1	107·6	113·1	118·1	124·1	128·3	137·2	140·8
100	99·33	102·9	106·9	111·7	118·5	124·3	129·6	135·8	140·2	149·4	153·2

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Tables of the Studentized Range, alpha= 0.10

Denominator df	Numerator											
	2	3	4	5	6	7	8	9	10	11	12	13
1	8.93	13.44	16.36	18.49	20.15	21.50	22.64	23.62	24.48	25.24	25.92	26.54
2	4.13	5.73	6.77	7.54	8.14	8.63	9.05	9.41	9.73	10.01	10.26	10.49
3	3.33	4.47	5.20	5.74	6.16	6.51	6.81	7.06	7.29	7.49	7.67	7.83
4	3.02	3.98	4.59	5.04	5.39	5.68	5.93	6.14	6.33	6.49	6.65	6.78
5	2.85	3.72	4.26	4.66	4.98	5.24	5.46	5.65	5.82	5.97	6.10	6.22
6	2.75	3.56	4.07	4.44	4.73	4.97	5.17	5.34	5.50	5.64	5.76	5.88
7	2.68	3.45	3.93	4.28	4.56	4.78	4.97	5.14	5.28	5.41	5.53	5.64
8	2.63	3.37	3.83	4.17	4.43	4.65	4.83	4.99	5.13	5.25	5.36	5.46
9	2.59	3.32	3.76	4.08	4.34	4.55	4.72	4.87	5.01	5.13	5.23	5.33
10	2.56	3.27	3.70	4.02	4.26	4.47	4.64	4.78	4.91	5.03	5.13	5.23
11	2.54	3.23	3.66	3.97	4.21	4.40	4.57	4.71	4.84	4.95	5.05	5.15
12	2.52	3.20	3.62	3.92	4.16	4.35	4.51	4.65	4.78	4.89	4.99	5.08
13	2.50	3.18	3.59	3.89	4.12	4.30	4.46	4.60	4.72	4.83	4.93	5.02
14	2.49	3.16	3.56	3.85	4.08	4.27	4.42	4.56	4.68	4.79	4.88	4.97
15	2.48	3.14	3.54	3.83	4.05	4.24	4.39	4.52	4.64	4.75	4.84	4.93
16	2.47	3.12	3.52	3.80	4.03	4.21	4.36	4.49	4.61	4.71	4.81	4.89
17	2.46	3.11	3.50	3.78	4.00	4.18	4.33	4.46	4.58	4.68	4.77	4.86
18	2.45	3.10	3.49	3.77	3.98	4.16	4.31	4.44	4.55	4.65	4.75	4.83
19	2.45	3.09	3.47	3.75	3.97	4.14	4.29	4.42	4.53	4.63	4.72	4.80
20	2.44	3.08	3.46	3.74	3.95	4.12	4.27	4.40	4.51	4.61	4.70	4.78
21	2.43	3.07	3.45	3.72	3	4.11	4.26	4.38	4.49	4.59	4.68	4.76
22	2.43	3.06	3.44	3.71	3.92	4.10	4.24	4.36	4.47	4.57	4.66	4.74
23	2.42	3.05	3.43	3.70	3.91	4.08	4.23	4.35	4.46	4.56	4.64	4.72
24	2.42	3.05	3.42	3.69	3.90	4.07	4.21	4.34	4.45	4.54	4.63	4.71
25	2.42	3.04	3.42	3.68	3.89	4.06	4.20	4.32	4.43	4.53	4.61	4.69
26	2.41	3.04	3.41	3.68	3.88	4.05	4.19	4.31	4.42	4.52	4.60	4.68
27	2.41	3.03	3.40	3.67	3.87	4.04	4.18	4.30	4.41	4.50	4.59	4.67
28	2.41	3.03	3.40	3.66	3.87	4.03	4.17	4.29	4.40	4.49	4.58	4.66
29	2.40	3.02	3.39	3.65	3.86	4.02	4.16	4.28	4.39	4.48	4.57	4.65
30	2.40	3.02	3.39	3.65	3.85	4.02	4.16	4.28	4.38	4.47	4.56	4.64
40	2.38	2.99	3.35	3.61	3.80	3.96	4.10	4.22	4.32	4.41	4.49	4.56
60	2.36	2.96	3.31	3.56	3.76	3.91	4.04	4.16	4.25	4.34	4.42	4.49
80	2.35	2.95	3.29	3.54	3.73	3.89	4.01	4.13	4.22	4.31	4.39	4.46
120	2.34	2.93	3.28	3.52	3.71	3.86	3.99	4.10	4.19	4.28	4.35	4.42
240	2.34	2.92	3.26	3.50	3.68	3.83	3.96	4.07	4.16	4.24	4.32	4.39
∞	2.33	2.90	3.24	3.48	3.66	3.81	3.93	4.04	4.13	4.21	4.29	4.35